## edexcel

Mark Scheme (Results)
Summer 2013

GCE Further Pure Mathematics 1 (6667/01)

## Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

## www.edexcel.com/contactus

## Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2013
Publications Code UA035965
All the material in this publication is copyright
© Pearson Education Ltd 2013

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.


## 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
-     - The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

## General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

1. Factorisation
$\left(x^{2}+b x+c\right)=(x+p)(x+q)$, where $|p q|=|c|$, leading to $\mathrm{x}=$
$\left(a x^{2}+b x+c\right)=(m x+p)(n x+q)$, where $|p q|=|c|$ and $|m n|=|a|$, leading to $\mathrm{x}=$
2. Formula

Attempt to use correct formula ( with values for $a, b$ and c).
3. Completing the square

Solving $x^{2}+b x+c=0: \quad\left(x \pm \frac{b}{2}\right)^{2} \pm q \pm c, \quad q \neq 0, \quad$ leading to $\mathrm{x}=\ldots$

## Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ( $\left.x^{n} \rightarrow x^{n-1}\right)$
2. Integration

Power of at least one term increased by 1 . $\left(x^{n} \rightarrow x^{n+1}\right)$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 1. | $\mathbf{M}=\left(\begin{array}{cc} x & x-2 \\ 3 x-6 & 4 x-11 \end{array}\right)$ |  |  |
|  | $\operatorname{det} \mathbf{M}=x(4 x-11)-(3 x-6)(x-2)$ | Correct attempt at determinant | M1 |
|  | $x^{2}+x-12(=0)$ | Correct 3 term quadratic | A1 |
|  | $(x+4)(x-3)(=0) \rightarrow \mathrm{x}=\ldots$ | Their 3TQ = 0 and attempts to solve relevant quadratic using factorisation or completing the square or correct quadratic formula leading to $x=$ | M1 |
|  | $x=-4, x=3$ | Both values correct | A1 |
|  |  |  | (4) |
|  |  |  | Total 4 |
| Notes |  |  |  |
|  | $x(4 x-11)=(3 x-6)(x-2)$ award first M1 |  |  |
|  | $\pm\left(x^{2}+x-12\right)$ seen award first M1A1 |  |  |
|  | Method mark for solving 3 term quadratic: <br> 1. Factorisation <br> $\left(x^{2}+b x+c\right)=(x+p)(x+q)$, where $\|p q\|=\|c\|$, leading to $\mathrm{x}=$ $\left(a x^{2}+b x+c\right)=(m x+p)(n x+q)$, where $\|p q\|=\|c\|$ and $\|m n\|=\|a\|$, leading to $\mathrm{x}=$ <br> 2. Formula <br> Attempt to use correct formula (with values for $a, b$ and $c$ ). <br> 3. Completing the square <br> Solving $x^{2}+b x+c=0: \quad\left(x \pm \frac{b}{2}\right)^{2} \pm q \pm c, \quad q \neq 0, \quad$ leading to $\mathrm{x}=\ldots$ |  |  |
|  | Both correct with no working 4/4, only one correct 0/4 |  |  |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 2 | $\mathrm{f}(\mathrm{x})=\cos \left(x^{2}\right)-x+3$ |  |  |
| (a) | $\begin{aligned} & \mathrm{f}(2.5)=1.499 \ldots . . \\ & \mathrm{f}(3)=-0.9111 \ldots . \end{aligned}$ | Either any one of $f(2.5)=$ awrt 1.5 or $f(3)=$ awrt -0.91 | M1 |
|  | Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore root or equivalent. | Both $f(2.5)=$ awrt 1.5 and $f(3)=$ awrt -0.91 , sign change and conclusion. | A1 |
|  | Use of degrees gives $f(2.5)=1.494$ and $f(3)=0.988$ which is awarded M1A0 |  | (2) |
| (b) | $\frac{3-\alpha}{" 0.91113026188 "}=\frac{\alpha-2.5}{" 1.4994494182 "}$ | Correct linear interpolation method accept equivalent equation - ensure signs are correct. | M1 A1ft |
|  | $\alpha=\frac{3 \times 1.499 \ldots+2.5 \times 0.9111 \ldots}{1.499 \ldots+0.9111 \ldots}$ |  |  |
|  | $\alpha=2.81$ (2d.p.) | cao | A1 |
|  |  |  | (3) |
|  |  |  | Total 5 |
| Notes | Alternative (b) |  |  |
|  | Gradient of line is $-\frac{\text { '1.499...'+'0.9111...' }}{0.5}(=-4.82)$ (3sf). Attempt to find equation of straight line and equate y to 0 award M1 and A1ft for their gradient awrt 3sf. |  |  |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 3(a) | Ignore part labels and mark part (a) and part (b) together. |  |  |
|  | $\mathrm{f}\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^{3}-9\left(\frac{1}{2}\right)^{2}+k\left(\frac{1}{2}\right)-13$ | Attempts f(0.5) | M1 |
|  | $\left(\frac{1}{4}\right)-\left(\frac{9}{4}\right)+\left(\frac{k}{2}\right)-13=0 \Rightarrow k=\ldots \ldots$ | Sets $\mathrm{f}(0.5)=0$ and leading to $k=$ | dM1 |
|  | $\mathrm{k}=30$ | cao | A1 |
|  | Alternative using long division: |  |  |
|  | $\begin{aligned} & 2 x^{3}-9 x^{2}+k x-13 \div(2 x-1) \\ & =x^{2}-4 x+\frac{1}{2} k-2 \text { (Quotient) } \\ & \text { Re mainder } \frac{1}{2} k-15 \end{aligned}$ | Full method to obtain a remainder as a function of $k$ | M1 |
|  | $\frac{1}{2} k-15=0$ | Their remainder $=0$ | dM1 |
|  | $k=30$ |  | A1 |
|  | Alternative by inspection: |  |  |
|  | $(2 x-1)\left(x^{2}-4 x+13\right)=2 x^{3}-9 x^{2}+30 x-13$ | First M for $(2 x-1)\left(x^{2}+b x+c\right)$ or $\left(x-\frac{1}{2}\right)\left(2 x^{2}+b x+c\right)$ <br> Second M1 for $a x^{2}+b x+c$ where $(b=-4 \text { or } c=13) \text { or }(b=-8 \text { or } c=26)$ | M1dM1 |
|  | $\mathrm{k}=30$ |  | A1 |
|  |  |  | (3) |
| (b) | $\begin{aligned} & \mathrm{f}(x)=(2 x-1)\left(x^{2}-4 x+13\right) \\ & \text { or }\left(x-\frac{1}{2}\right)\left(2 x^{2}-8 x+26\right) \end{aligned}$ | M1: $\left(x^{2}+b x \pm 13\right)$ or ( $2 x^{2}+b x \pm 26$ ) Uses inspection or long division or compares coefficients and $(2 x-1)$ or $\left(x-\frac{1}{2}\right)$ to obtain a quadratic factor of this form. | M1 |
|  | $x^{2}-4 x+13$ or $2 x^{2}-8 x+26$ | A1 $\left(x^{2}-4 x+13\right)$ or $\left(2 x^{2}-8 x+26\right)$ seen | A1 |
|  | $x=\frac{4 \pm \sqrt{4^{2}-4 \times 13}}{2}$ or equivalent | Use of correct quadratic formula for their 3TQ or completes the square. | M1 |
|  | $x=\frac{4 \pm 6 i}{2}=2 \pm 3 i$ | oe | A1 |
|  |  |  | (4) |
|  |  |  | Total 7 |


| Question <br> Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 4(a) | $y=\frac{4}{x}=4 x^{-1} \Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=-4 x^{-2}=-\frac{4}{x^{2}}$ | $\frac{\mathrm{d} y}{\mathrm{~d} x}=k x^{-2}$ | M1 |
|  | $x y=4 \Rightarrow x \frac{\mathrm{~d} y}{\mathrm{~d} x}+y=0$ | Use of the product rule. The sum of two terms including $\mathrm{d} y / \mathrm{d} x$, one of which is correct. |  |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} t} \cdot \frac{\mathrm{~d} t}{\mathrm{~d} x}=-\frac{2}{t^{2}} \cdot \frac{1}{2}$ | their $\frac{\mathrm{d} y}{\mathrm{~d} t} \times\left(\frac{1}{\text { their } \frac{\mathrm{d} x}{\mathrm{~d} t}}\right)$ |  |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=-4 x^{-2} \text { or } x \frac{\mathrm{~d} y}{\mathrm{~d} x}+y=0 \text { or } \frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{2}{t^{2}} \cdot \frac{1}{2}$ <br> or equivalent expressions | Correct derivative $-4 x^{-2},-\frac{y}{x}$ or $\frac{-1}{t^{2}}$ | A1 |
|  | So, $m_{N}=t^{2}$ | Perpendicular gradient rule $m_{N} m_{T}=-1$ | M1 |
|  | $y-\frac{2}{t}=t^{2}(x-2 t)$ | $\begin{aligned} & y-\frac{2}{t}=\text { their } m_{N}(x-2 t) \text { or } \\ & y=m x+c \text { with their } m_{N} \text { and }\left(2 t, \frac{2}{t}\right) \text { in } \end{aligned}$ <br> an attempt to find ' $c$ '. <br> Their gradient of the normal must be different from their gradient of the tangent and have come from calculus and should be a function of $t$. | M1 |
|  | $t y-t^{3} x=2-2 t^{4} *$ |  | A1* cso |
|  |  |  | (5) |
| (b) | $t=-\frac{1}{2} \Rightarrow-\frac{1}{2} y-\left(-\frac{1}{2}\right)^{3} x=2-2\left(-\frac{1}{2}\right)^{4}$ | Substitutes the given value of $t$ into the normal | M1 |
|  | $4 y-x+15=0$ |  |  |
|  | $\begin{aligned} & y=\frac{4}{x} \Rightarrow x^{2}-15 x-16=0 \text { or } \\ & \left(2 t, \frac{2}{t}\right) \rightarrow \frac{8}{t}-2 t+15=0 \Rightarrow 2 t^{2}-15 t-8=0 \text { or } \\ & x=\frac{4}{y} \Rightarrow 4 y^{2}+15 y-4=0 . \end{aligned}$ | Substitutes to give a quadratic | M1 |
|  | $\begin{aligned} & (x+1)(x-16)=0 \Rightarrow x=\text { or } \\ & (2 \mathrm{t}+1)(t-8)=0 \Rightarrow t=\text { or } \\ & (4 y-1)(y+4)=0 \Rightarrow y= \end{aligned}$ | Solves their 3TQ | M1 |
|  | $(P: x=-1, y=-4)(Q:) x=16, y=\frac{1}{4}$ | Correct values for $x$ and $y$ | A1 |
|  |  |  | (4) |
|  |  |  | Total 9 |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 5(a) | $(r+2)(r+3)=r^{2}+5 r+6$ |  | B1 |
|  | $\sum\left(r^{2}+5 r+6\right)=\frac{1}{6} n(n+1)(2 n+1)+5 \times \frac{1}{2} n(n+1),+6 n$ | M1: Use of correct expressions for $\sum r^{2}$ and $\sum r$ <br> B1ft: $\sum k=n k$ | M1,B1ft |
|  | $=\frac{1}{3} n\left[\frac{1}{2}(n+1)(2 n+1)+\frac{15}{2}(n+1)+18\right]$ | M1:Factors out $n$ ignoring treatment of constant. <br> A1: Correct expression with $\frac{1}{3} n$ or $\frac{1}{6} n$ factored out, allow recovery. | M1 A1 |
|  | $\begin{aligned} & \left(=\frac{1}{3} n\left[n^{2}+\frac{3}{2} n+\frac{1}{2}+\frac{15}{2} n+\frac{15}{2}+18\right]\right) \\ & =\frac{1}{3} n\left[n^{2}+9 n+26\right]^{*} \end{aligned}$ | Correct completion to printed answer | A1*cso |
|  |  |  | (6) |
| 5(b) | $\sum_{r=n+1}^{3 n}=\frac{1}{3} 3 n\left((3 n)^{2}+9(3 n)+26\right)-\frac{1}{3} n\left(n^{2}+9 n+26\right)$ | M1: $\mathrm{f}(\mathbf{3 n})-\mathrm{f}(n$ or $n+1)$ and attempt to use part (a). A1: Equivalent correct expression | M1A1 |
|  | $3 \mathrm{f}(\boldsymbol{n})-\mathrm{f}(n$ or $n+1)$ is M0 |  |  |
|  | $\left(=n\left(9 n^{2}+27 n+26\right)-\frac{1}{3} n\left(n^{2}+9 n+26\right)\right)$ |  |  |
|  | $=\frac{2}{3} n\left(\frac{27}{2} n^{2}+\frac{81}{2} n+39-\frac{1}{2} n^{2}-\frac{9}{2} n-13\right)$ | Factors out $=\frac{2}{3} n$ dependent on previous M1 | dM1 |
|  | $=\frac{2}{3} n\left(13 n^{2}+36 n+26\right)$ | Accept correct expression. | A1 |
|  | $(a=13, b=36, c=26)$ |  |  |
|  |  |  | (4) |
|  |  |  | Total 10 |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 6(a) | $y=2 a^{\frac{1}{2}} x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=a^{\frac{1}{2}} x^{-\frac{1}{2}}$ | $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ | M1 |
|  | $y^{2}=4 a x \Rightarrow 2 y \frac{\mathrm{~d} y}{\mathrm{~d} x}=4 a$ | $k y \frac{\mathrm{~d} y}{\mathrm{~d} x}=c$ |  |
|  | $\text { or } \frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} t} \cdot \frac{\mathrm{~d} t}{\mathrm{~d} x}=2 a \cdot \frac{1}{2 a p}$ | $\frac{\mathrm{d} y}{\mathrm{~d} t} \times \frac{1}{\frac{\mathrm{~d} x}{\mathrm{~d} t}}$. Can be a function of $p$ or $t$. |  |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=a^{\frac{1}{2}} x^{-\frac{1}{2}} \text { or } 2 y \frac{\mathrm{~d} y}{\mathrm{~d} x}=4 a \text { or } \frac{\mathrm{d} y}{\mathrm{~d} x}=2 a \cdot \frac{1}{2 a p}$ | Differentiation is accurate. | A1 |
|  | $y-2 a p=\frac{1}{p}\left(x-a p^{2}\right)$ | Applies $y-2 a p=$ their $m\left(x-a p^{2}\right)$ or $y=($ their $m) x+c$ using $x=a p^{2}$ and $y=2 a p$ in an attempt to find $c$. Their $\boldsymbol{m}$ must be a function of $\boldsymbol{p}$ from calculus. | M1 |
|  | $p y-x=a p^{2} *$ | Correct completion to printed answer* | A1 cso |
|  |  |  | (4) |
| (b) | $q y-x=a q^{2}$ |  | B1 |
|  |  |  | (1) |
| (c) | $q y-a q^{2}=p y-a p^{2}$ | Attempt to obtain an equation in one variable $x$ or $y$ | M1 |
|  | $\begin{aligned} & y(q-p)=a q^{2}-a p^{2} \\ & y=\frac{a q^{2}-a p^{2}}{q-p} \end{aligned}$ | Attempt to isolate $x$ or $y$ | M1 |
|  | $\begin{aligned} & y=a(p+q) \text { or } a p+a q \\ & x=a p q \end{aligned}$ | A1: Either one correct simplified coordinate <br> A1: Both correct simplified coordinates | A1,A1 |
|  | (R(apq, ap $+a q)$ ) |  |  |
|  |  |  | (4) |
| (d) | ' apq ' $=-a$ | Their $x$ coordinate of $R=-a$ | M1 |
|  | $p q=-1$ | Answer only: Scores $2 / 2$ if $x$ coordinate of $R$ is apq otherwise $0 / 2$. | A1 |
|  |  |  | (2) |
|  |  |  | Total 11 |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 7 | $z_{1}=2+3 i, \quad z_{2}=3+2 i$ |  |  |
| (a) | $z_{1}+z_{2}=5+5 \mathrm{i} \Rightarrow\left\|z_{1}+z_{2}\right\|=\sqrt{5^{2}+5^{2}}$ | Adds $\mathrm{z}_{1}$ and $\mathrm{z}_{2}$ and correct use of Pythagoras. i under square root award M0. | M1 |
|  | $\sqrt{50}(=5 \sqrt{2})$ |  | A1 cao |
|  |  |  | (2) |
| (b) | $\begin{aligned} & \frac{z_{1} z_{3}}{z_{2}}=\frac{(2+3 i)(a+b i)}{3+2 i} \\ & =\frac{(2+3 i)(a+b i)(3-2 i)}{(3+2 i)(3-2 i)} \end{aligned}$ | Substitutes for $\mathrm{z}_{1}, \mathrm{z}_{2}$ and $\mathrm{z}_{3}$ and multiplies by $\frac{3-2 i}{3-2 i}$ | M1 |
|  | $(3+2 i)(3-2 i)=13$ | 13 seen. | B1 |
|  | $\frac{z_{1} z_{3}}{z_{2}}=\frac{(12 a-5 b)+(5 a+12 b) \mathrm{i}}{13}$ | M1: Obtains a numerator with 2 real and 2 imaginary parts. | dM1A1 |
|  |  | A1: As stated or $\frac{(12 a-5 b)}{13}+\frac{(5 a+12 b)}{13} \mathrm{i}$ ONLY. |  |
|  |  |  | (4) |
| (c) | $\begin{aligned} & 12 a-5 b=17 \\ & 5 a+12 b=-7 \end{aligned}$ | Compares real and imaginary parts to obtain 2 equations which both involve $a$ and $b$. Condone sign errors only. | M1 |
|  | $\begin{aligned} & 60 a-25 b=85 \\ & 60 a+144 b=-84 \end{aligned} \Rightarrow b=-1$ | Solves as far as $a=$ or $b=$ | dM1 |
|  | $a=1, b=-1$ | Both correct | A1 |
|  |  | Correct answers with no working award 3/3. |  |
|  |  |  | (3) |
| (d) | $\arg (w)=-\tan ^{-1}\left(\frac{7}{17}\right)$ | Accept use of $\pm \tan ^{-1}$ or $\pm \tan$. awrt $\pm 0.391$ or $\pm 5.89$ implies M1. | M1 |
|  | =awrt -0.391 or awrt 5.89 |  | A1 |
|  |  |  | (2) |
|  |  |  | Total 11 |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 8(a) | $\mathbf{A}^{2}=\left(\begin{array}{cc} 6 & -2 \\ -4 & 1 \end{array}\right)\left(\begin{array}{cc} 6 & -2 \\ -4 & 1 \end{array}\right)=\left(\begin{array}{cc} 44 & -14 \\ -28 & 9 \end{array}\right)$ | M1:Attempt both $\mathbf{A}^{2}$ and 7A + 2I | M1A1 |
|  | $7 \mathbf{A}+2 \mathbf{I}=\left(\begin{array}{cc} 42 & -14 \\ -28 & 7 \end{array}\right)+\left(\begin{array}{ll} 2 & 0 \\ 0 & 2 \end{array}\right)=\left(\begin{array}{cc} 44 & -14 \\ -28 & 9 \end{array}\right)$ | A1: Both matrices correct |  |
|  | OR $\mathbf{A}^{2}-7 \mathbf{A}=\mathbf{A}(\mathbf{A}-7 \mathbf{I})$ | M1 for expression and attempt to substitute and multiply $(2 \times 2)(2 \times 2)=2 \times 2$ |  |
|  | $\mathbf{A}(\mathbf{A}-7 \mathbf{I})=\left(\begin{array}{cc}6 & -2 \\ -4 & 1\end{array}\right)\left(\begin{array}{ll}-1 & -2 \\ -4 & -6\end{array}\right)=\left(\begin{array}{ll}2 & 0 \\ 0 & 2\end{array}\right)=2 \mathbf{I}$ | A1 cso |  |
|  |  |  | (2) |
| (b) | $\mathbf{A}^{2}=7 \mathbf{A}+2 \mathbf{I} \Rightarrow \mathbf{A}=7 \mathbf{I}+2 \mathbf{A}^{\mathbf{- 1}}$ | Require one correct line using accurate expressions involving $\mathbf{A}^{-1}$ and identity matrix to be clearly stated as I . | M1 |
|  | $\mathbf{A}^{-1}=\frac{1}{2}(\mathbf{A}-7 \mathbf{I})^{*}$ |  | A1* cso |
|  | Numerical approach award 0/2. |  |  |
|  |  |  | (2) |
| (c) | $\mathbf{A}^{-1}=\frac{1}{2}\left(\begin{array}{ll}-1 & -2 \\ -4 & -6\end{array}\right)$ | Correct inverse matrix or equivalent | B1 |
|  | $\frac{1}{2}\left(\begin{array}{ll}-1 & -2 \\ -4 & -6\end{array}\right)\binom{2 k+8}{-2 k-5}=\frac{1}{2}\binom{-2 k-8+4 k+10}{-8 k-32+12 k+30}$ | Matrix multiplication involving their inverse and $k$ : $(2 \mathrm{x} 2)(2 \mathrm{x} 1)=2 \mathrm{x} 1$ <br> N.B. <br> $\left(\begin{array}{cc}6 & -2 \\ -4 & 1\end{array}\right)\binom{2 k+8}{-2 k-5}$ is M0 | M1 |
|  | $\binom{k+1}{2 k-1}$ or $(k+1,2 k-1)$ | $(k+1)$ first A1, $(2 k-1)$ second A1 | A1,A1 |
|  | Or: |  |  |
|  | $\left(\begin{array}{cc}6 & -2 \\ -4 & 1\end{array}\right)\binom{x}{y}=\binom{2 k+8}{-2 k-5}$ | Correct matrix equation. | B1 |
|  | $\begin{aligned} & 6 x-2 y=2 k+8 \\ & -4 x+y=-2 k-5 \Rightarrow x=\ldots \text { or } y=\ldots \end{aligned}$ | Multiply out and attempt to solve simultaneous equations for $x$ or $y$ in terms of $k$. | M1 |
|  | $\binom{k+1}{2 k-1}$ or $(k+1,2 k-1)$ | $(k+1)$ first A1, $(2 k-1)$ second A1 | A1,A1 |
|  |  |  | (4) |
|  |  |  | Total 8 |


| Question Number | Scheme | Notes | Marks |
| :---: | :---: | :---: | :---: |
| 9(a) | $\begin{aligned} & u_{1}=8 \text { given } \\ & n=1 \Rightarrow u_{1}=4^{1}+3(1)+1=8 \quad(\therefore \text { true for } n=1) \end{aligned}$ | $4^{1}+3(1)+1=8$ seen | B1 |
|  | Assume true for $n=k$ so that $u_{k}=4^{k}+3 k+1$ |  |  |
|  | $u_{k+1}=4\left(4^{k}+3 k+1\right)-9 k$ | Substitute $u_{k}$ into $u_{k+1}$ as $u_{k+1}=4 u_{k}-9 k$ | M1 |
|  | $=4^{k+1}+12 k+4-9 k=4^{k+1}+3 k+4$ | Expression of the form $4^{k+1}+a k+b$ | A1 |
|  | $=4^{k+1}+3(k+1)+1$ | Correct completion to an expression in terms of $k+1$ | A1 |
|  | If true for $n=k$ then true for $n=k+1$ and as true for $n=1$ true for all $n$ | Conclusion with all 4 underlined elements that can be seen anywhere in the solution; $n$ defined incorrectly award A0. | A1 cso |
|  |  |  | (5) |
| (b) | Condone use of $\boldsymbol{n}$ here. |  |  |
|  | $\begin{aligned} & \text { lhs }=\left(\begin{array}{ll} 3 & -4 \\ 1 & -1 \end{array}\right)^{1}=\left(\begin{array}{ll} 3 & -4 \\ 1 & -1 \end{array}\right) \\ & r h s=\left(\begin{array}{cc} 2(1)+1 & -4(1) \\ 1 & 1-2(1) \end{array}\right)=\left(\begin{array}{ll} 3 & -4 \\ 1 & -1 \end{array}\right) \end{aligned}$ | Shows true for $m=1$ | B1 |
|  | Assume $\left(\begin{array}{ll}3 & -4 \\ 1 & -1\end{array}\right)^{k}=\left(\begin{array}{cc}2 k+1 & -4 k \\ k & 1-2 k\end{array}\right)$ |  |  |
|  | $\left(\begin{array}{ll}3 & -4 \\ 1 & -1\end{array}\right)^{k+1}=\left(\begin{array}{cc}2 k+1 & -4 k \\ k & 1-2 k\end{array}\right)\left(\begin{array}{cc}3 & -4 \\ 1 & -1\end{array}\right)$ | $\left(\begin{array}{ll} 3 & -4 \\ 1 & -1 \end{array}\right)\left(\begin{array}{cc} 2 k+1 & -4 k \\ k & 1-2 k \end{array}\right)$ <br> award M1 | M1 |
|  | $=\left(\begin{array}{cc}6 k+3-4 k & -8 k-4+4 k \\ 3 k+1-2 k & -4 k-1+2 k\end{array}\right)$ | Or equivalent 2 x 2 matrix. $\left(\begin{array}{cc} 6 k+3-4 k & -12 k-4+8 k \\ 2 k+1-k & -4 k-1+2 k \end{array}\right)$ <br> award A1from above. | A1 |
|  | $=\left(\left(\begin{array}{cc}2 k+3 & -4 k-4 \\ k+1 & -2 k-1\end{array}\right)\right)$ |  |  |
|  | $=\left(\begin{array}{cc}2(k+1)+1 & -4(k+1) \\ k+1 & 1-2(k+1)\end{array}\right)$ | Correct completion to a matrix in terms of $k+1$ | A1 |
|  | If true for $m=k$ then true for $m=k+1$ and as true for $m=1$ true for all $m$ | Conclusion with all 4 underlined elements that can be seen anywhere in the solution; $m$ defined incorrectly award A0. | A1 cso |
|  |  |  | (5) |
|  |  |  | Total 10 |

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623467467
Fax 01623450481
Email publication.orders@edexcel.com
Order Code UA035965 Summer 2013
 Welsh Assembly Government
For more information on Edexcel qualifications, please visit our website www.edexcel.com

